

Adv. Math Decision Making

Unit 1: Analyzing Numerical Data



MAtTHew M. Winking

Enrico Fermi (1901 – 1954) was an Italian physicist that worked in the United States. He was known for his contributions in creating the first nuclear reactor and his work on the Manhattan Project (creating the atomic bomb). He received the Nobel Prize for his work in Physics in 1938. Fermi was also well known for this ability to make good estimates of large quantities with little actual data and using only a few logical assumptions. When determining these approximations, he would usually first try to determine reasonable lower bound and upper bound limits of the estimate. Then, he would make logical assumptions and work towards finding a reasonable estimate (usually by some power of 10) within the earlier defined limits. A question he would commonly pose while lecturing at the University of Chicago was, “How many piano tuners are there in the city of Chicago?” Today, making such estimates of large quantities are commonly referred to as Fermi Problems.



Let’s try a few.

How many Skittles?

How many skittles candies can fit in a mason jar that has the inner dimensions that are approximately a cylinder with a height of 9 cm and a base radius of 3 cm?
(Reminder: Cylinder’s Volume = $\pi \cdot r^2 \cdot h$).



1. What would you decide is a reasonable **lower limit** for the number of skittles in the jar? (i.e. a number of skittles that you are certain there are at least that many in the jar)
2. What would your group decide is a reasonable **upper limit** for the number of skittles in the jar? (i.e. a number of skittles that must be more than the maximum amount of skittles in the jar)

3. What would you decide is a reasonable **lower limit** for the number of skittles in the container?



4. What would your group decide is a reasonable **upper limit** for the number of skittles in the container?

Assume that the container is roughly a rectangular prism such that Rectangular Prism Volume = $l \cdot w \cdot h$

5. What would you decide is a reasonable **lower limit** for the number of blocks in the container?

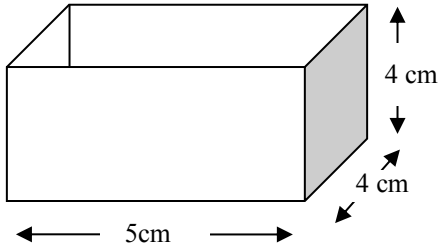


6. What would your group decide is a reasonable **upper limit** for the number of blocks in the container?

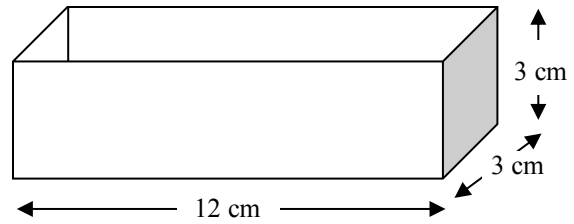
For a little more precision in our estimates we can run some trials and use empirical ratios

7. Consider the investigational data found by filling the following containers with skittles.

A few students conducted a study of some solids and found the following container (solid #1) held 192 skittles.



A few students conducted a study of some solids and found the following container (solid #2) held 258 skittles.



Fill out the following table:

	<i>Solid #1</i>	<i>Solid #2</i>
<i>Number of Skittles :</i>		
<i>Volume:</i>		
<i>Ratio $\left(\frac{\text{Skittles}}{\text{cm}^3}\right)$:</i>		

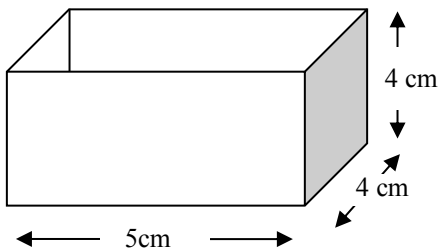
Using the empirical ratios above determine more precisely, how many skittles candies can fit in a mason jar that has the inner dimensions that are approximately a cylinder with a height of 9 cm and a base radius of 3 cm?

(Reminder: Cylinder's Volume = $\pi \cdot r^2 \cdot h$).

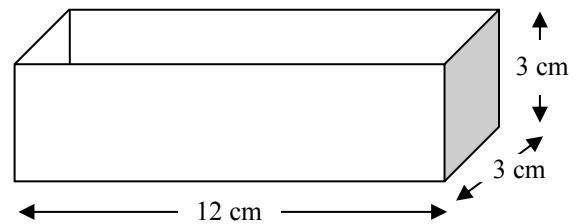


8. Consider the investigational data found by filling the following containers with skittles.

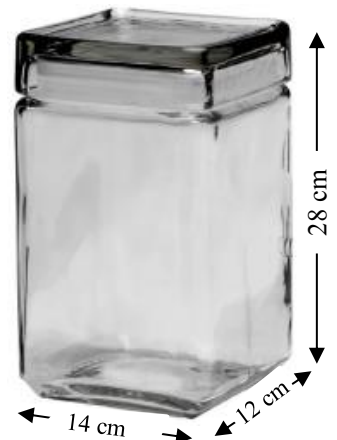
A few students conducted a study of some solids and found the following container (solid #1) held 32 jellybeans.



A few students conducted a study of some solids and found the following container (solid #2) held 43 jellybeans.



Using the empirical data approximate how many jellybeans the candy jar at the right will hold if it is approximately a rectangular prism with the inner dimensions 12 cm by 14 cm by 28 cm.



A student was conducting a study to determine how many pages he would need for the book he is writing. So, he found that the following number of words fit on each type of the following papers using an 11 point font:

Paper Size	Average Number of Words
8.5 inches by 11 inches	800
5.5 inches by 6.5 inches	300
12 inches by 18 inches	1900

- Using the collected data above what would be a reasonable rough estimate of the number of words per square inch of writing paper using an 11 point font?
- Roughly, how many pages would his book be if his novel was approximately 65,000 words and he used a paper size of 5 inches by 7 inches with an 11 point font (i.e. how many page numbers would the book require just for the novel)?

A student was conducting a study to determine how many loose Navel oranges that could be packed in different box sizes at the local farmer's market. The average navel orange had a diameter of 3.2 inches.

Box Dimensions	Number of Oranges in the Box
20 inch x 13 inch x 12 inch	96 oranges
12 inch x 12 inch x 12 inch	53 oranges
20 inch x 10 inch x 6 inch	37 oranges

- Using the collected data above what would be a reasonable rough estimate of a ratio of how many cubic inches per orange?
- Given the Farmer's Market wanted to use a box that had the dimensions, $16 \text{ in.} \times 9 \text{ in.} \times 9 \text{ in.}$, approximately how many oranges should fit in the box based on the information provided by the table?

- On a highway a wreck occurred and caused an 10 mile traffic jam on one side of the road. The average car is 13.5 feet in length, the average truck is 20 feet in length, and the average 18 wheeler tractor trailer is 75 feet in length. 70% of the traffic jam consists of cars, 20% trucks, and 10% 18 wheeler tractor trailers. If the average distance between vehicles is 3 feet, how many vehicles are stuck in the traffic jam?



14. A person just purchased the vending machine shown. Each compartment has the dimensions of 5 inches by 7 inches by 17 inches. Assuming the vending machine uses Gumballs that are approximately spherical and 1 inch in diameter, how many gumballs should fit in one of the compartments?

(Hint: Packing spheres in a rectangular prism usually take up 190% of the volume of the spheres.)

SHOW YOUR WORK



15. A person is using a new tennis ball launching machine that is 15 inches by 15 inches by 14 . Assuming the machine uses tennis balls that are spherical and 2.7 inches in diameter, how many tennis balls should fit in one of the top of the machine?

(Hint: Packing spheres in a rectangular prism usually take up 190% of the volume of the spheres using random packing.)

SHOW YOUR WORK



Estimating the number of people in a large crowd (for example watching a parade or attending/marching in a political rally) is quite challenging and often leads to controversies. One method sometimes used is to focus on a small section of the crowd, such as a rectangular area.



16. Mark off a 5 foot by 5 foot square, and see how many people can comfortably stand inside the square as if they are at an outdoor concert.

a. How many people fit?

b. Divide the area of the square by the number of people that fit inside the square and explain what this ratio means.

c. Use the ratio to estimate the size of a crowd that is 10 feet deep on both sides of a street for a mile and is filled with people.

17. A football field is 360 feet long and 160 feet wide. The principal is making an evacuation plan for the school. How many students can the principal expect to fit on the football field in an emergency? (Remember the expected floor space a standing person occupies is about 2.5 sq feet)



1. Justine is a wedding coordinator. She is selecting the menu options for the reception.
 - First, she will give the guests a choice of Salad or Soup for the first course.
 - Then, she will allow them to pick from a chicken, steak, or a vegetarian for their main course.
 - Finally, she will allow the guests to choose bride’s vanilla cake or groom’s chocolate cake.



Create a tree diagram showing every plausible dinner a guest could select:

How many outcomes are possible?

Show how you could use the counting principle to determine the number of outcomes.

2. At a New Car Dealership, a particular model comes in 4 different trim levels (CX, DX, EX, and Si). The same model comes in 5 different colors (Night Black, Pearl White, Evening Blue, Sandy Red, and Forest Green). The model of car also has 3 different interior options (Grey Cloth, Tan Cloth, Black Leather). How many different versions of this model can be created from these options?



3. A seven digit telephone number is of the form ABC-DEFG. In one particular state, the digit ‘A’ is restricted to any number between 1 and 9. The digits B and C are restricted to any number between 2 and 9. The digits D,E,F, and G have no restriction. How many seven digit phone numbers are possible with these restrictions?



4. A ten digit telephone number is of the form (XYZ) – ABC –DEFG. In one particular state, there are 4 possible area codes (202, 341, 602, and 581). The digit ‘A’ is restricted to a number 2 through 8. The digits B and C can be any number but they cannot repeat. The digits D,E,F, and G have no restriction. How many seven digit phone numbers are possible with these restrictions?



5. How many area codes of the form (XYZ) are possible if the digit 'X' and 'Y' can be any number 1 through 9 and the digit 'Z' can be any number 2 through 9?



6. A seven digit telephone number is of the form ABC-DEFG. In one particular state, the digit 'A' can be any digit except 0 and 1. The digits B and C can be any digit from 2 - 9. The digits D, E, F, and G can be any digit 0 – 9 except they can't all be the same (e.g. 0000, 1111, 2222,etc.). How many seven digit phone numbers are possible with these restrictions?



7. A student number for a high school requires that student identification number consist of 6 characters. The first 4 characters can be any number without restriction. The last 2 characters are letters and cannot repeat. How many student ID's are possible?



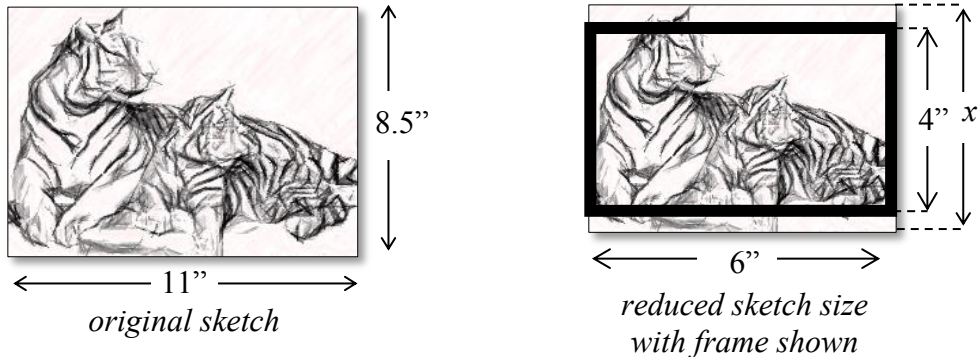
8. A lock on a fence door has a 3 digit combination. Each digit can be any number between 1 – 8. The only restriction is that all 4 characters cannot be the same (e.g. 111, 222, 333, etc.). How many combinations are possible?



9. A suitcase has a lock on it consisting of four numbers. Each number could be any number 0-9. The only restriction is that two numbers in a row cannot be the same (e.g. you couldn't use 3224 because the 2's are adjacent but you could use 3434 since none of the same numbers are adjacent).

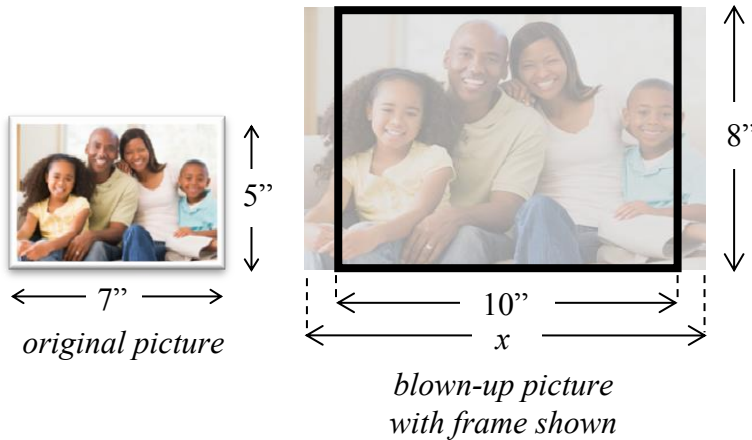


1. A student sketched some art on an 8.5 inch x 11 inch piece of paper. He wants to resize it to fit an 4 inch x 6 inch frame (as shown below)



What percent of the original sketch was still able to be included in the frame?

2. A person wants to increase a (5 in. x 7 in.) photo to an (8 in. x 10 in.) but since the aspect ratios are not the same some of the picture will get chopped off.

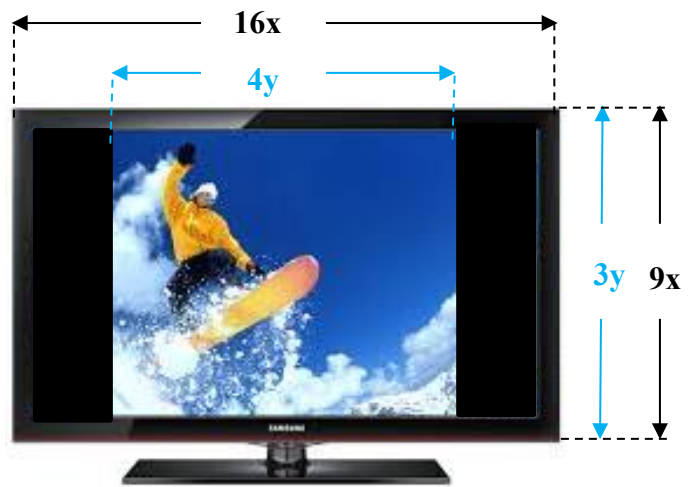


What percentage of the picture can be used in the (8 in. x 10 in.) frame?

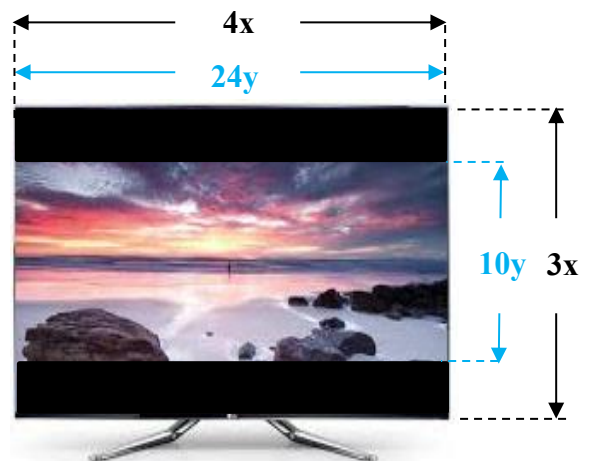
3. Television sizes are usually described by the length of their diagonal measure. What would be the listed size of the TV shown in the picture?



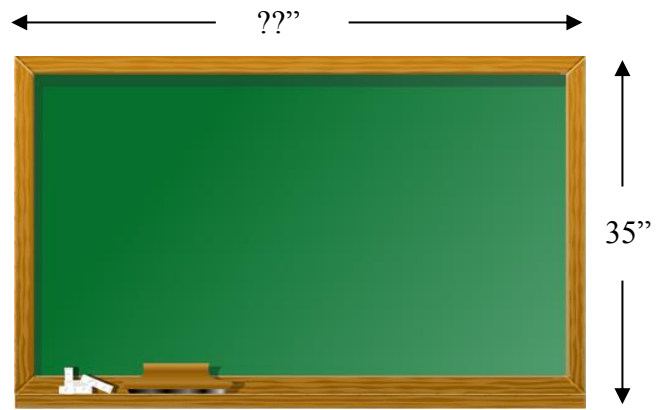
4. A monitor with a screen dimension ratio of 16:9 is playing a video image with a dimension ratio of 4:3 at its fullest size which left a pillar-boxed image. What percent of the screen's area is occupied by the image?



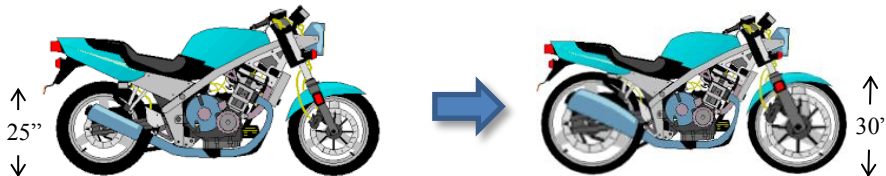
5. A monitor with a screen dimension ratio of 4:3 is playing a video image with a dimension ratio of 24:10 at its fullest size which left a letter-boxed image. What percent of the screen's area is occupied by the image?



6. A chalk board's dimensions have an aspect ratio of 5:3 (i.e. W:H). If the height of the board is 35 inches how wide is the board?



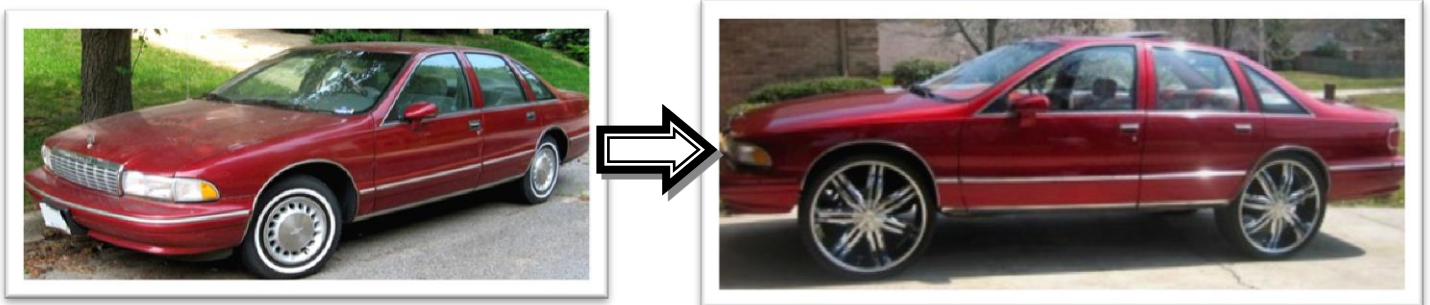
7. A person changed the size of the wheels on his motorcycle



He changed the size from a tire that had a 25 inch diameter to a tire that has a 30 inch diameter. He didn't realize this would affect his speedometer and odometer. He didn't recalibrate either after changing the wheel size. How far did he really travel on the new tires, if his odometer shows he traveled 1200 miles?

How fast is he really going on the new tires when his speedometer reads 60 mph?

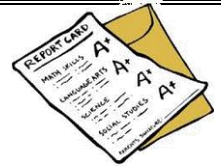
11. A person decided to alter the appearance of his car by adding a lift kit, new suspension, & larger wheels. The original 1993 Chevrolet Caprice had P225/75R15 tires. The modified version had P305/30R26 tires.



a. Determine the diameter of the original tires.

b. Determine the diameter of the new tires.

- c. If the speedometer wasn't recalibrated, how fast is the person actually going on the new tires when his speedometer shows 60 mph?



When a weighted average is applied to a set of numbers, more importance (weight) is placed on some components of the set. Your final average in this class I probably an example of a weighted average.

Consider two grading systems

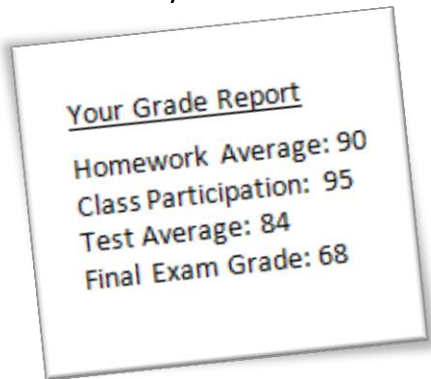


Mr. Tats Grade Weighting	
Homework Average	25%
Class Participation	10%
Test Average	40%
Final Exam Grade	25%

Mrs. Etercsid Grade Weighting	
Homework Average	15%
Class Participation	10%
Test Average	60%
Final Exam Grade	15%



1. If you had the following grade shown below, determine what your grade would be with each teacher.



2. Use the following information to find your course average in Mr. Tats class:

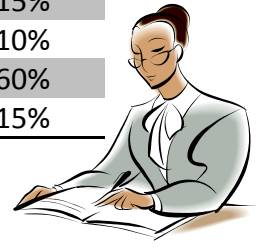
- Homework {83, 92, 95, 90}
- Class Participation {90, 100}
- Test Grades {90, 78, 84, 88}
- Final Exam Grade {84}

3. If Mr. Tats allowed you to retake your final exam, what score would you have to get on the final exam to make at least a 90% in the class?



Mr. Tats Grade Weighting	
Homework Average	25%
Class Participation	10%
Test Average	40%
Final Exam Grade	25%

Mrs. Etercsid Grade Weighting	
Homework Average	15%
Class Participation	10%
Test Average	60%
Final Exam Grade	15%



4. The following shows a student's grade in Ms. Etercsid's class. Determine the minimum grade need on the final exam to make at least a 70% in the class.
- Homework {70, 82, 64}
 - Class Participation {70, 80}
 - Test Grades {60, 68, 62, 74}
 - Final Exam Grade {??}

Other weighted averages

5. A new cars salesperson, sold an average of \$28,000 per day for the first 5 days she worked. The next 3 days she worked, she sold an average of \$42,000. How much did she sell on average per day over the entire 8 days?
6. Kelly works two jobs. She usually works at a retail store for 7 hours during the day and is paid \$12 per hour by the store. In the evening she works at a restaurant as a manager for 5 hours and is paid \$20 per hour. How much does she make on average per hour for the day?
7. James worked for 12 consecutive days, earning an average wage of \$160 per day as a contractor. During the first 6 days, James averaged \$140 per day and his average wage for the next 5 days was \$175. How much did he earn on the last day?



Slugging Percentage

In baseball there are many common statistics used batting average and slugging percentage. Batting average is just simply the percentage of the number of "at bats" for which the player gets a hit. If a player has 20 hits in 50 at bats then their batting average would be, $\frac{20}{50} = 0.400$. Their slugging percentage however, weights the type of hit where doubles count double, triples count triple, and homeruns count quadruple and would be given by the formula:



$$\text{Slugging Percentage} = \frac{(\text{singles}) + 2 \cdot (\text{doubles}) + 3 \cdot (\text{triples}) + 4 \cdot (\text{home runs})}{(\text{at bats})}$$

8. Calculate Freddie's Slugging Percentage if he has 42 singles, 12 doubles, 4 triples, and 14 home runs in 230 at bats.



9. Can you use the above information to determine Freddie's current batting average?

10. If Andrelton has a slugging percentage of 0.450 and has 12 doubles, 3 triples, and 8 homeruns in 200 at bats then how many singles must he have?

11. Can you determine Andrelton's current batting average?

Universal Product Codes (UPCs), typically in the form of barcodes

Identification numbers are present everywhere in society. Today's identification numbers are more sophisticated than those introduced years earlier (for example, Social Security numbers). Today's numbers have a check digit to partially ensure that they have been correctly scanned or entered into a computer.

Universal Product Codes (UPCs), typically in the form of barcodes, identify retail products.

The 12-digit UPC barcode consists of three parts:

- manufacturer number,
- product number, and
- check digit.

For example, the manufacturer number for the Dr. Pepper Company is 078000 and appears in the first six digits of all of the company's product UPC barcodes. GS1, formerly the Uniform Code Council, issues a company this six-digit number. Every item sold by a company requires a different five-digit product number. This includes specific products, their different sizes, their array of colors, their variety of flavors, and other distinguishing features. The last number is the check digit, which guards against entry errors and fraud. The check digit in a UPC number (that is, the twelfth digit) is determined in the following manner:



- Multiply the first digit by 3.
- Add the second digit.
- Multiply the third digit by 3.
- Add the fourth digit.
- Continue this alternating process for the Digits 5 to 12.

The check digit is chosen so that the calculation described previously totals a number whose final digit is 0. In the UPC number $a_1a_2a_3a_4a_5a_6a_7a_8a_9a_{10}a_{11}d$, the check digit is d , for which the sum

$$3a_1 + a_2 + 3a_3 + a_4 + 3a_5 + a_6 + 3a_7 + a_8 + 3a_9 + a_{10} + 3a_{11} + d$$

ends in 0. In this weighted sum, the weights are: {3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1}.

When entering a code number, the single-digit error is most common (for example, keying in 8 instead of 3). Another common error is the transposition error, where the order of two adjacent digits is reversed (for example, writing 83 instead of 38). Systems have been established to detect and correct (when possible) these and other errors almost immediately.

1. Show that **0-58200-48826-5** is a valid UPC number.
2. Show that **0-52200-48826-5** is an invalid UPC number.
3. Determine the check digit (**d**) for the UPC number 3-81370-09213-**d**.

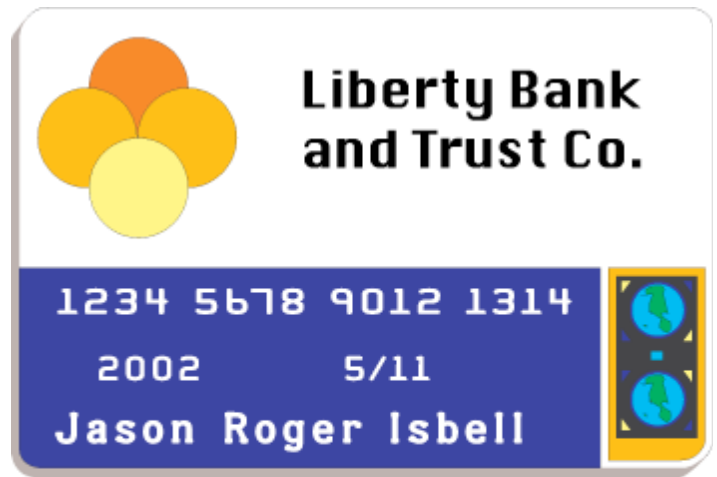
Credit Cards

Identification numbers are present everywhere in society. Today's identification numbers are more sophisticated than those introduced years earlier (for example, Social Security numbers). Today's numbers have a check digit to partially ensure that they have been correctly scanned or entered into a computer.

Credit cards have 16-digit numbers, of which the first 15 digits identify the credit card and the sixteenth digit is the check digit. The following figure shows the significance of the digits:

MII stands for major industry identifier; 4 and 5 indicate "Banking and Financial." VISA cards begin with 4 and MasterCard cards with 5.

1. MasterCard numbers begin with 51, 52, 53, 54, or 55. What is the maximum number of credit cards that MasterCard can issue?



1	2	3	4	5	6	7	8	9	0	1	2	1	3	1	4
Mill	Issuer				Account				Check						
	Identifier				Number				Digit						

A **check digit** is used to help validate credit card numbers. The credit card companies use the Codabar method to determine the check digit. This method consists of the following steps:

- Add the digits in the odd-numbered positions and double this total.
- Add the number of odd-position digits that are more than 4 to the total.
- Add the even-position digits.
- Choose a check digit that makes this calculation total a number whose final digit is 0.

Libraries, shipping/receiving companies, and blood banks also use the Codabar method.

2. Show that the check digit (d) for the VISA card 4162 0012 3456 789d is 3.
3. What is the check digit (d) for the MasterCard number 5424 9813 2720 008d?
4. Show that 4128 0012 4389 0110 is an invalid VISA credit card number.